Large random matching markets with localized preference structures can exhibit large cores[2]

Problem: When a *matching market* supports multiple stable matchings, core-selecting matching algorithms can be manipulated.

Question: When do random markets support many core matchings?[1]

Received wisdom: Cores are generically small in random markets. Conjectured by Roth and Peranson (1999), proved for limited preference lists (Immorlica and Mahdian (2005), Kojima and Pathak (2009)) and for imbalanced markets (Ashlagi, Kanoria, and Leshno (2015)).

Limited counterexamples are non-robust: isolated replicas (Immorlica and Mahdian (2005)), knife-edge balanced markets (Ashlagi *et al.* (2015))

[1] Conventionally, core size is measured in fraction of agents with > 1 stable match.

[2] These slides present independent work and only the opinions of the named author.

`http://static.rossry.net/papers/working/pref-structure-core_20200625.pdf`

Central result: Random markets support non-vanishing cores when agents' preferences have localized structure.

- *Locality:* If 1 prefers A and B, then 2 prefers $B \Rightarrow$ 2 likely prefers A.
- Standard feature of random network models.

Prior "core convergence" results need homogeneity in agents' preferences.[3]

Working model: Arrange colleges in a circle. Students draw a location, then draw preferences from the nearest k colleges.

Result: For fixed k, core size is bounded below as #agents grows.

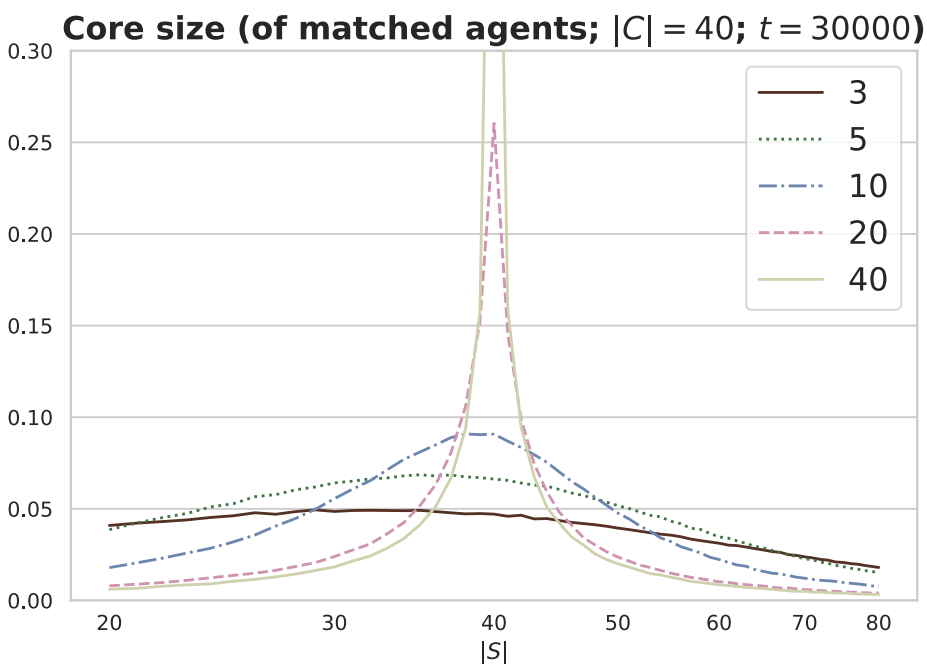
- Proof by a constructive lower bound on event frequency.[4]

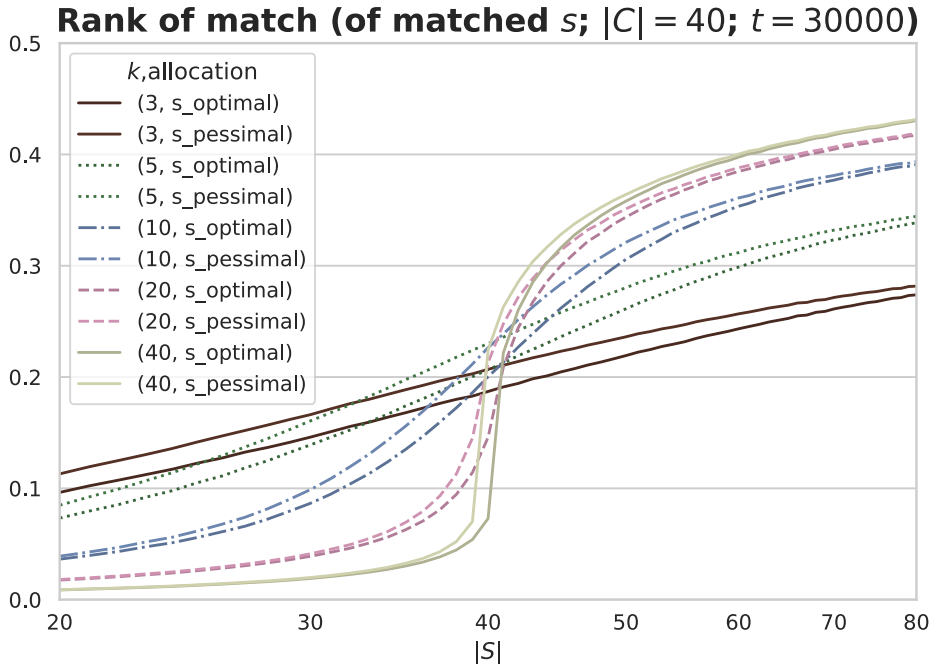
[3] While homogeneous correlation induces core convergence (Ashlagi et al. (2015)), localizing correlation supports non-vanishing cores – even in large, imbalanced markets.

[4] Proof technique adapted in part from Hassidim, Romm, and Shorrer (2019).

Result: Under preference locality, core size can be material in simulation.

- *(Left)* Core size of homogeneous (yellow) vs localized (brown) random markets as imbalance varies from $|S| \ll |C|$ to $|S| \gg |C|$.





Result: Under preference locality, the gap in agents' welfare between their optimal and pessimal stable match can be material in simulation.

- *(Right)* Gap between students' optimal- and pessimal-match rank remains as imbalance varies in localized (brown) random markets.

- Theoretical results extend to a general model of preference locality.
  - Satisfied by many random models for structured networks.
- Small cores observed in practice can be explained by other factors:
  - (Localized) general popularity
  - Search costs and diversified portfolios (see Shorrer (2020))
  - Strong *localized imbalance* between sides
  - Existing manipulation of market outcomes (by accepting side)